Mansoura University
Faculty of Engineering
Mechanical Power Eng. Dept.

Computer Application in MPE4325
3rd year
Time :3 hr

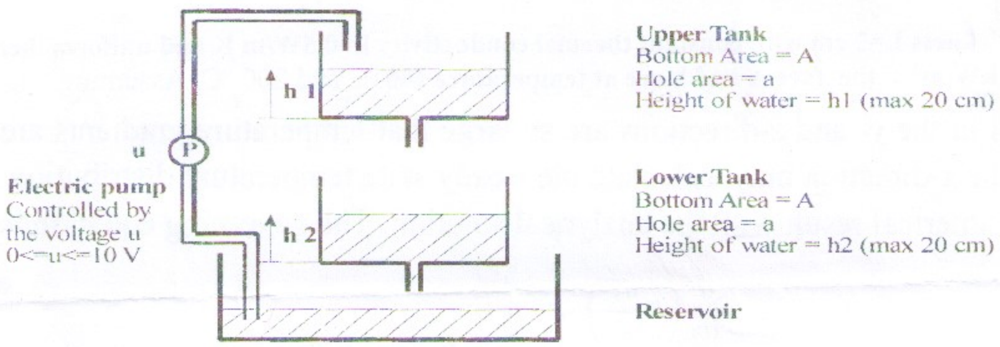
Attempt to solve all Questions and write a flow chart for all program

Question (1) [15]

1-a) Use Runge Kutta fourth order method to integrate $y' = 4e^{0.8t} - 0.5y$ from t=0 to 4 with step size of 1. The initial condition at t=0 is y=2. Note that the exact solution can be determined analytically as

$$y = \frac{4}{1.3}(e^{0.8t} - e^{-0.5t}) + 2e^{-0.5t}$$

1-b) The models are used for basic courses in control theory and it is possible to visualize different control aspects. Since the models are simple and normally known for the students they are also used as examples in other courses. This text how the will therefore describe how the models are used in the course "MATLAB programming and simulation". The model of the water tank system is shown in the figure below:



The equations for the system:

$$\frac{dh1}{dt} = \beta * u - \alpha\sqrt{2*g*h1}\ , \qquad \frac{dh2}{dt} = \alpha\sqrt{2*g*h1} - \alpha\sqrt{2*g*h2}$$

Where $\beta = \frac{km}{A}$; $\alpha = \frac{a}{A}$, km = Pump constant = $2.7*e^{-6}$, A = Bottom area = $2734*e^{-6}\ m^2$,

a = Hole area = $7*e^{-6}\ m^2$, g = Gravity constant = $9.82\ m/s^2$

Question (2) [15]

$$2\frac{d^2U}{dx^2} = \frac{dU}{dt}$$

U(x, t)

for $0 \le x \le \pi$, $0 \le t \le 0.2$ with the initial/boundary conditions : $u(x, 0) = \sin(2x)$, $u(0, t) = 0$, $u(\pi, t) = 0$

(i) By substituting $u(x, t) = \sin(2x)e^{-8t}$ into the above equation, verify that this is a solution to the PDE.

(ii) With the solution region divided into $M \times N = 40 \times 100$ sections, does the explicit forward Euler method converge? What is the value of $r = A\Delta t/(Dx)^2$?

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(iii) If you increase N (the number of subintervals along the t-axis) to 125 for improving the numerical stability, does the explicit forward Euler method converge? What is the value of $r = A\Delta t/(Dx)^2$? Use the MATLAB statements in the following box to find the maximum absolute errors of the numerical solutions obtained by the three methods. Which method yields the smallest error?

Question (3) [15]

Consider the source free heat conduction in an insulated rod whose ends are maintained at constant temperature of 100 °C and 500 °C respectively.

$$\frac{d}{dx}\left(k\frac{dT}{dx}\right) = 0$$

Calculate the steady state temperature distribution in the rod. Thermal conductivity equals 100 W/m K cross section of the rod A=10X10^{-3} and the rod length is 0.5m

Question (4) [15]

The large plate of thickness L=2 cm with constant thermal conductivity k=0.5W/m K and uniform heat generation q=1000 kW/m^3 . the face A and b are at temperature 100 °C and 200 °C. Assuming the the dimensions in the y- and z-directions are so large that temperature gradients are significant in the x-direction only, calculate the steady state temperature distribution. Compare the numerical result with the analytical solution. The governing equation is

$$\frac{d}{dx}\left(k\frac{dT}{dx}\right) - q = 0$$

Question (5) [15]

A property ϕ is transported by means of convection and diffusion through the one-dimensional domain sketched in Figure 5.2. The governing equation and the boundary conditions are $\phi_0 = 1$ at $x = 0$ and $\phi_L = 0$ at $x = L$. Using five equally spaced cells and the central differencing scheme for convection and diffusion calculate the distribution of ϕ as a function of x for (i) Case 1: $u = 0.1$ m/s, (ii) Case 2: $u = 2.5$ m/s, and compare the results with the analytical solution

$$\frac{\phi - \phi_0}{\phi_L - \phi_0} = \frac{\exp(\rho u x/\Gamma) - 1}{\exp(\rho u L/\Gamma) - 1}$$

(iii) Case 3: recalculate the solution for $u = 2.5$ m/s with 20 grid nodes and compare the results with the analytical solution. The following data apply: length $L = 1.0$ m, $\rho = 1.0$ kg/m^3, $\Gamma = 0.1$ kg/m/s.

Good Luck
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Solve all problem

1) Write a computer program, flow chart and mathematical solution of The free-fall velocity of a parachutist can be simulated as

$$\frac{dv}{dt} = g - \frac{c_d}{m} v^2$$

where v = velocity(m/s), t = time (s), g = acceleration due to gravity (9.81 m/s^2), c_d = drag coefficient (kg/m), and m = mass (kg). For a 80-kg parachutist, solve this equation using

   a) Euler's Method
   b) Huen's Method(Improvement of Euler's Method)
   c) Higher Order Runge Kutta's Method

from t = 0 to 30 s given that v(0) = 0 with step size, h = 2. During free fall, c_d = 0.25 kg/m.

2) Consider the following set of coupled first order, nonlinear ODEs

$$\frac{dx}{dt} = x + y - x(x^2 + y^2)$$

$$\frac{dy}{dt} = -x + y - y(x^2 + y^2)$$

   a) Solve the set of equations with initial conditions x(0) = 2 and y(0) = 2 over the time interval $0 \le t \le 20$. Plot x vs t and y vs t in two different figures. Use **hold on** to keep the plots and graphs of subsequent solutions as overlay plots. Save the solutions x and y in, say x1 and y1 for later use.

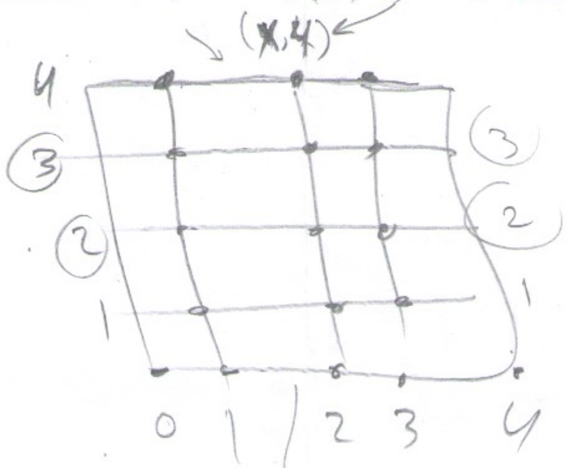
3) Laplace's Equation—Steady-State Temperature Distribution.Consider Laplace's equation

$$\nabla^2 u(x, y) = \frac{\partial^2 u(x, y)}{\partial x^2} + \frac{\partial^2 u(x, y)}{\partial y^2} = 0 \qquad \text{for } 0 \le x \le 4, 0 \le y \le 4$$

with the boundary conditions

$$u(0, y) = e^y - \cos y, \qquad u(4, y) = e^y \cos 4 - e^4 \cos y$$

$$u(x, 0) = \cos x - e^x, \qquad u(x, 4) = e^4 \cos x - e^x \cos 4$$



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Consider the following set of coupled first order, nonlinear ODEs

$$\frac{dx}{dt} = x + y -- x(x^2 + y^2)$$

$$\frac{dy}{dt} = -x + y - y(x^2 + y^2)$$

b) Solve the set of equations with initial conditions x(0) = 2 and y(0) = 2 over the time interval $0 \leq t \leq 20$. Plot x vs t and y vs t in two different figures. Use **hold on** to keep the plots and graphs of subsequent solutions as overlay plots. Save the solutions x and y in, say x1 and y1 for later use.

c) Solve the given set of equations for the following set of initial conditions and plot the solutions as above (over-layed on the old solutions). (I) $(x_o, y_o) = (1,0)$ (ii) $(x_o, y_o) = (0.1,0)$ and (iii) $(x_o, y_o) = (0, 1)$. Save the solution in each case for later use. Do the plots suggest a particular long term behaviour of the system?

d) The ultimate periodic solution that all solutions seem to get attracted to is called a **limit cycle**. It is much easier to visualise the limit cycle in phase space, that is, the plane of x and y. Plot x versus y for all solutions on the same figure. You should clearly see the limit cycle on this phase plot. Make sure the axis have equal lengths

e) Try solutions with some other initial conditions.